

Optimal Controlled Teleportation

Ting Gao^{1,2}, Feng-Li Yan^{2,3}, and You-Cheng Li³

¹ *College of Mathematics and Information Science,
Hebei Normal University, Shijiazhuang 050016, China*

² *Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Str. 1, D-85748 Garching, Germany*

³ *College of Physics and Information Engineering,
Hebei Normal University, Shijiazhuang 050016, China*

(Dated: February 2, 2008)

We give the analytic expressions of maximal probabilities of successfully controlled teleporting an unknown qubit via every kind of tripartite states. Besides, another kind of localizable entanglement is also determined. Furthermore, we give the sufficient and necessary condition that a three-qubit state can be collapsed to an EPR pair by a measurement on one qubit, and characterize the three-qubit states that can be used as quantum channel for controlled teleporting a qubit of unknown information with unit probability and with unit fidelity.

PACS numbers: 03.67.Hk, 89.70.+c

Quantum teleportation is commonly considered as one of the most striking progress of quantum information theory. In the seminal work of Bennett *et al.* [1], they showed that an arbitrary unknown state of a qubit could be teleported from a sender Alice to a spatially distant receiver Bob with the aid of long-range Einstein-Podolsky-Rosen (EPR) correlations and the transmission of two bits of classical information. Since then, quantum teleportation has been developed by many authors due to its important applications in quantum communication and quantum computation. At present, teleportation has been generalized to various cases [2, 3, 4, 5, 6, 7, 8, 9, 10]. On the other hand, in past several years quantum teleportation has been also experimentally demonstrated by several groups [11, 12].

The controlled quantum teleportation scheme was presented by Karlsson and Bourennane [6], with very similar ideas also in the quantum secret sharing paper of Hillery *et al.* [13]. In [6, 13] the entanglement property of the Greenberger-Horne-Zeilinger (GHZ) state is utilized for teleporting a qubit of unknown information. According to the scheme, a third side is included, so that the quantum channel is supervised by this additional side. An unknown state can be perfectly transported from one place to another place via previously shared quantum resource—GHZ state—by means of local operations and classical communications (LOCC) under the permission of the third party. The signal state can not be transmitted unless all three sides agree to cooperate. The controlled quantum teleportation is useful in the context of quantum information such as networked quantum information processing and cryptographic conferencing [14, 15, 16], and controlled quantum secure direct communication and has other interesting applications, such as in opening account on the agreement of managers in a network. Recently, a number of works on controlled quantum teleportation have also been proposed [7, 8, 10], where they restrict themselves to the special quantum channels, such as GHZ state or W state. If a nonmaximally entangled state is taken as quantum channel, then

one can not teleport a qubit with unit probability and unit fidelity. However, it is possible to teleport a qubit with a probability $p < 1$, which is called probabilistic quantum teleportation [9].

The entanglement property lies at the very heart of quantum information theory. The reason is that entanglement is the physical resource to perform some of the most important quantum information tasks, such as quantum teleportation, quantum computation etc. In [18], Verstraete, Popp, and Cirac introduced a new concept which they called localizable entanglement (LE). This quantity not only has a very well defined physical meaning that treats entanglement as a truly physical resource, but also establishes a very close connection between entanglement and correlation functions. The LE of two particles is the maximal amount of entanglement that can be localized in these two particles, on average, by doing local measurement on the rest of the particles. The determination of the LE is a formidable task since it involves optimization over all possible local measurement strategies, and thus can not be determined in general. However, Verstraete, Popp, and Cirac gave tight upper bound and lower bound.

In this paper, we investigate the general case of controlled quantum teleportation — i.e. controlled teleporting a qubit of unknown information from a sender to a remote receiver via the control of a third agent by the use of a general three-qubit state — and its maximal successful probability, which is a kind of LE, but different from that in [18]. We give the analytic expression of the maximal successful probability and the exact value of LE in [18] for tripartite. Moreover, the sufficient and necessary condition that a general three-qubit states can collapsed to an EPR pair with certain probability by means of measurement on one qubit are given. In addition, we show in detail that for any given three-qubit state, how to choose measurement basis to achieve maximal successful probability of controlled teleportation. More surprising is the fact that there exist states that can not be converted to GHZ states under LOCC and can be used for perfect

controlled teleportation — controlled teleporting a qubit with unit fidelity and unit probability. In deed, we show that any tripartite entangled state can be used for perfect controlled teleportation if and only if it is LOCC equivalent to the following state

$$a_0|000\rangle + a_1|100\rangle + \frac{1}{\sqrt{2}}|111\rangle, \quad (1)$$

where $a_0 \geq 0$, $a_1 \geq 0$, and $a_0^2 + a_1^2 = \frac{1}{2}$. Here the first qubit is a controlled one.

Acín et al [17] proved that for every pure state of a composite system, 123, there exist orthonormal states $|0\rangle_1, |1\rangle_1$ for system 1, orthonormal states $|0\rangle_2, |1\rangle_2$ for system 2, and orthonormal states $|0\rangle_3, |1\rangle_3$ for system 3 such that

$$\begin{aligned} |\Psi\rangle_{123} = & a_0|000\rangle_{123} + a_1e^{i\mu}|100\rangle_{123} + a_2|101\rangle_{123} \\ & + a_3|110\rangle_{123} + a_4|111\rangle_{123}, \quad (2) \\ & a_i \geq 0, \quad 0 \leq \mu \leq \pi, \quad \sum_{i=0}^4 a_i^2 = 1. \end{aligned}$$

Suppose that Alice is to deliver an unknown state $|\psi\rangle_4 = \alpha|0\rangle_4 + \beta|1\rangle_4$ ($|\alpha|^2 + |\beta|^2 = 1$) to a distant receiver Bob supervised by the controller Charlie via a quantum channel of a normalized general pure three-qubit state in (2), where particle 1 belongs to Charlie, particle 2 is in Alice's side, while Bob has particle 3. Note that $a_0 \neq 0$. Since if $a_0 = 0$, then $|\Psi\rangle_{123}$ is a tensor product state of a pure state of particle 1 and a pure state of particles 2 and 3, but not a true tripartite entangled state.

After getting the approval of Charlie, Alice and Bob begin their teleportation under the control of Charlie.

The controller Charlie measures his particle in the measurement basis

$$\begin{aligned} |x\rangle &= \cos(\theta/2)|0\rangle + e^{i\varphi} \sin(\theta/2)|1\rangle, \\ |x\rangle^\perp &= \sin(\theta/2)|0\rangle - e^{i\varphi} \cos(\theta/2)|1\rangle, \end{aligned} \quad (3)$$

and broadcasts his measurement result to Alice and Bob. Here $\theta \in [0, \pi]$, $\varphi \in [0, 2\pi]$.

The tripartite state $|\Psi\rangle_{123}$ can be reexpressed as

$$|\Psi\rangle_{123} = \sqrt{p_1}|x\rangle_1|\Phi_1\rangle_{23} + \sqrt{p_2}|x\rangle_1^\perp|\Phi_2\rangle_{23}. \quad (4)$$

Here

$$\begin{aligned} p_1 &= \sin^2(\theta/2) + a_0^2 \cos \theta + a_0 a_1 \cos(\mu - \varphi) \sin \theta, \\ p_2 &= \cos^2(\theta/2) - a_0^2 \cos \theta - a_0 a_1 \cos(\mu - \varphi) \sin \theta, \\ |\Phi_1\rangle_{23} &= p_1^{-\frac{1}{2}} \{ [a_0 \cos(\theta/2) + a_1 e^{i(\mu-\varphi)} \sin(\theta/2)] |00\rangle_{23} \\ &\quad + e^{-i\varphi} \sin(\theta/2) [a_2 |01\rangle_{23} + a_3 |10\rangle_{23} + a_4 |11\rangle_{23}] \}, \\ |\Phi_2\rangle_{23} &= p_2^{-\frac{1}{2}} \{ [a_0 \sin(\theta/2) - a_1 e^{i(\mu-\varphi)} \cos(\theta/2)] |00\rangle_{23} \\ &\quad - e^{-i\varphi} \cos(\theta/2) [a_2 |01\rangle_{23} + a_3 |10\rangle_{23} + a_4 |11\rangle_{23}] \}. \end{aligned}$$

After Charlie's measurement, the quantum channel is collapsed to $|\Phi_1\rangle_{23}$, and $|\Phi_2\rangle_{23}$ with probability p_1 , and p_2 , respectively.

By Schmidt decomposition, there is

$$|\Phi_1\rangle_{23} = \sqrt{\lambda_{10}}|0'_2 0'_3\rangle + \sqrt{\lambda_{11}}|1'_2 1'_3\rangle, \quad (5)$$

$$|\Phi_2\rangle_{23} = \sqrt{\lambda_{20}}|\bar{0}_2 \bar{0}_3\rangle + \sqrt{\lambda_{21}}|\bar{1}_2 \bar{1}_3\rangle, \quad (6)$$

where $\{0'_2, 1'_2\}$ and $\{\bar{0}_2, \bar{1}_2\}$ ($\{0'_3, 1'_3\}$, and $\{\bar{0}_3, \bar{1}_3\}$) are orthonormal bases of system 2 (system 3), and

$$\begin{aligned} \lambda_{10} &= \frac{1 - \sqrt{1 - C_1^2}}{2}, \quad \lambda_{11} = \frac{1 + \sqrt{1 - C_1^2}}{2}, \\ \lambda_{20} &= \frac{1 - \sqrt{1 - C_2^2}}{2}, \quad \lambda_{21} = \frac{1 + \sqrt{1 - C_2^2}}{2}, \quad (7) \\ C_1 &= \frac{|a_0 a_4 e^{-i\varphi} \sin \theta + 2(a_1 a_4 e^{i\mu} - a_2 a_3) e^{-2i\varphi} \sin^2 \frac{\theta}{2}|}{p_1}, \\ C_2 &= \frac{|a_0 a_4 e^{-i\varphi} \sin \theta - 2(a_1 a_4 e^{i\mu} - a_2 a_3) e^{-2i\varphi} \cos^2 \frac{\theta}{2}|}{p_2}. \end{aligned}$$

Then, Alice makes a Bell measurement on her particles 2 and 4, and conveys her measurement outcome to Bob by transmitting two classical bits of information over a classical communication channel.

In order to achieve teleportation, Bob needs to introduce an auxiliary particle b with the initial state $|0\rangle_b$ and performs a collective unitary on the state of particles 3 and b . Then the measurement on his auxiliary particle b follows. If his measurement result is $|0\rangle_b$, Bob can fix up the state of his particle 3, recovering $|\psi\rangle$, by applying an appropriate local unitary operation. The achievable successful probability of teleporting the unknown state via $|\Phi_1\rangle_{23}$ ($|\Phi_2\rangle_{23}$) is $2\lambda_{10}$ ($2\lambda_{20}$).

Probability p of successfully controlled teleporting an unknown qubit using the state in (2) is

$$p = 2p_1\lambda_{10} + 2p_2\lambda_{20} = 1 - R(\theta, \varphi). \quad (8)$$

Here

$$\begin{aligned} R &= R(\theta, \varphi) = \sqrt{P(\theta, \varphi)} + \sqrt{Q(\theta, \varphi)}, \\ P(\theta, \varphi) &= p_1^2(1 - C_1^2), \quad Q(\theta, \varphi) = p_2^2(1 - C_2^2). \end{aligned} \quad (9)$$

It is known that if there exists (θ_0, φ_0) such that $P(\theta_0, \varphi_0) = 0$ or $Q(\theta_0, \varphi_0) = 0$, then Alice and Bob share an EPR pair with some finite probability by Charlie's measurement in the basis (3) with $(\theta, \varphi) = (\theta_0, \varphi_0)$. Furthermore, if there exists (θ_0, φ_0) such that $P(\theta_0, \varphi_0) = Q(\theta_0, \varphi_0) = 0$, then an EPR pair occurs with certainty after Charlie's measurement in the basis (3) with $(\theta, \varphi) = (\theta_0, \varphi_0)$. The tripartite state (2) with the property that both $P(\theta, \varphi)$ and $Q(\theta, \varphi)$ are equal to zero at the same point (θ, φ) can be used for perfect teleportation.

We first investigate the condition of $P(\theta, \varphi) = 0$.

$P(\theta, \varphi) = 0 \iff |\Phi_1\rangle_{23}$ is a Bell state \iff the concurrence C_1 of $|\Phi_1\rangle_{23}$ is 1.

Since $P(0, \varphi) = a_0^2 \neq 0$, so we suppose $\theta \in (0, \pi]$. Let $|\phi_1\rangle_{23} = \frac{ae^{i\alpha}|00\rangle_{23} + a_2|01\rangle_{23} + a_3|10\rangle_{23} + a_4|11\rangle_{23}}{\sqrt{a^2 + a_2^2 + a_3^2 + a_4^2}}$. Here $ae^{i\alpha} \equiv ta_0e^{i\varphi} + a_1e^{i\mu}$, and $t = \cot \frac{\theta}{2}$, while a is the absolute of complex number $ae^{i\alpha}$, α is the argument of $ae^{i\alpha}$. Note that the concurrence C_1 of $|\Phi_1\rangle_{23}$ is equal to the concurrence $C(|\phi_1\rangle_{23})$ of $|\phi_1\rangle_{23}$, and $C(|\phi_1\rangle_{23}) = 1$ iff $[C(|\phi_1\rangle_{23})]^2 = 1$. $[C(|\phi_1\rangle_{23})]^2 = 1$ means that

$$(a^2 - a_4^2)^2 + 2(aa_2 - a_3a_4)^2 + 2(aa_3 - a_2a_4)^2 + 8aa_2a_3a_4(1 + \cos \alpha) + (a_2^2 - a_3^2)^2 = 0. \quad (10)$$

From Eq.(10), we see that $C_1 = 1$ if and only if $a_2 = a_3$, $a = a_4$, and $\alpha = \pi$. Using these equalities, we obtain the

following results: If $a_1 = 0$, then $\varphi = \pi$ and $t = \frac{a_4}{a_0}$. If $\mu = 0$, then $\varphi = \pi$ and $t = \frac{a_1+a_4}{a_0}$. If $\mu = \pi$, then either $\varphi = 0$ and $t = \frac{a_1-a_4}{a_0}$ in case of $a_1 > a_4$ or $\varphi = \pi$ and $t = \frac{a_4-a_1}{a_0}$ in case of $a_1 < a_4$. If $a_1 \sin \mu \neq 0$, then $\cot \frac{\theta}{2} = \frac{\sqrt{a_1^2+2a_1a_4\cos\mu+a_4^2}}{a_0}$, $\cot \varphi = \cot \mu + \frac{a_4}{a_1 \sin \mu}$, $\varphi \in (\pi, 2\pi)$.

Similarly, we derive that $Q(\theta, \varphi) = 0$ if and only if the coefficients of the tripartite state in (2) satisfy $a_2 = a_3$.

Thus, we find that a three-qubit state in (2) can be collapsed to an EPR pair with certain probability by a measurement on the first qubit if and only if $a_2 = a_3$.

Next we characterize the states such that both $P(\theta, \varphi)$ and $Q(\theta, \varphi)$ are equal to 0 at the same point (θ, φ) .

By above discussion, we need only to find the condition such that $Q(\theta_0, \varphi_0) = 0$ for each case with $P(\theta_0, \varphi_0) = 0$. Note that $a_0 \neq 0$. We see that $Q(\theta_0, \varphi_0) = (1 - 2a_3^2 - 2a_4^2)^2 + 4a_3^2(a_1 - a_4)^2 = 0$ if and only if $a_2 = a_3 = 0$ and $a_4 = \frac{1}{\sqrt{2}}$ in case of $a_1 \sin \mu = 0$ and $a_2 = a_3$. Similarly, we have that when $a_2 = a_3$, and $a_1 \sin \mu \neq 0$, $Q(\theta_0, \varphi_0) = (1 - 2a_3^2 - 2a_4^2)^2 + 4a_3^2(a_1^2 + 2a_1a_4\cos\mu + a_4^2) = 0$ if and only if $a_2 = a_3 = 0$ and $a_4 = \frac{1}{\sqrt{2}}$.

Therefore, three-qubit state in the generalized Schmidt decomposition (2) can be used for perfect teleportation if and only if it is the state (1).

Now we investigate how to achieve the maximum of probability of successfully controlled teleporting an unknown qubit state via an arbitrary partially entangled quantum channel (2).

Obviously, the maximum of (8) is

$$p_{\max} = \max\{p\} = 1 - \min\{R(\theta, \varphi)\} = 1 - R_{\min}. \quad (11)$$

In order to reach the maximal probability of exact controlled teleportation, the supervisor Charlie needs only to choose optimal measurement basis, i.e. he selects θ_0 and φ_0 such that $R_{\min} = R(\theta_0, \varphi_0)$.

Note that the minimum of $R(\theta, \varphi)$ should occur at the points such that $P(\theta, \varphi) = 0$, $Q(\theta, \varphi) = 0$, and

$$R'_\theta(\theta, \varphi) = \frac{\partial R}{\partial \theta} = 0, \quad R'_\varphi(\theta, \varphi) = \frac{\partial R}{\partial \varphi} = 0. \quad (12)$$

Combining these two equations gives

$$\frac{\partial P}{\partial \theta} \frac{\partial Q}{\partial \varphi} - \frac{\partial Q}{\partial \theta} \frac{\partial P}{\partial \varphi} = 0, \quad (13)$$

$$P \left(\frac{\partial Q}{\partial \varphi} \right)^2 - Q \left(\frac{\partial P}{\partial \varphi} \right)^2 = 0, \quad (14)$$

$$P \left(\frac{\partial Q}{\partial \theta} \right)^2 - Q \left(\frac{\partial P}{\partial \theta} \right)^2 = 0. \quad (15)$$

Let us look at the general case — the quantum channel with parameters satisfying $a_0a_1a_2a_3a_4\sin\mu \neq 0$. Suppose $\sin\theta \neq 0$, and $P(\theta, \varphi)Q(\theta, \varphi) \neq 0$.

By (13), there is

$$2b_1x^2 + (b_2\cos\varphi + b_3\sin\varphi)x - a_0^2b_1 + b_4\cos 2\varphi + b_5\sin 2\varphi = 0, \quad (16)$$

where

$$\begin{aligned} x &= a_0 \cot \theta, \quad g_1 = a_2a_3a_4, \\ g_2 &= a_2^2a_3^2 - (a_2^2 + a_3^2)a_4^2, \quad g_3 = 2a_1^2 + 2a_4^2 - 1, \\ b_1 &= a_1g_1\sin\mu, \quad b_2 = 2a_1\sin\mu(3a_1g_1\cos\mu - g_2), \\ b_3 &= 2a_1g_2\cos\mu + g_1(1 - 2a_0^2 - a_1^2 - 3a_1^2\cos 2\mu - 2a_4^2), \\ b_4 &= a_1\sin\mu[g_1(a_0^2 - 1 + 4a_1^2\cos^2\mu + 2a_4^2) - 2a_1g_2\cos\mu], \\ b_5 &= g_1^2 - a_1g_1[a_1^2\cos 3\mu - (a_2^2 + a_3^2 - a_4^2)\cos\mu] + a_1^2g_2\cos 2\mu. \end{aligned}$$

Eq.(14) subtracted from Eq.(15) is

$$8d_1x^3 + 4x^2(d_2\sin\varphi + d_3\cos\varphi) + 2x(d_4 + d_5\cos 2\varphi + d_6\sin 2\varphi) + d_7\cos\varphi + d_8\sin\varphi + d_9\cos 3\varphi + d_{10}\sin 3\varphi = 0, \quad (17)$$

where

$$\begin{aligned} d_1 &= a_1g_1(g_3 + a_0^2)\cos\mu - a_1^2g_2 - g_1^2, \\ d_2 &= 2a_1\sin\mu[a_1g_1(2g_3 + a_0^2)\cos\mu - 2g_1^2 + (a_0^2 - 2a_1^2)g_2], \\ d_3 &= g_1(3a_0^2 - 1 - 2a_0^4 + 4a_1^2a_4^2 + 4a_2^2a_3^2 - 2a_0^2a_4^2 \\ &\quad + 4a_1^2g_3\cos^2\mu - 2a_0^2a_1^2\sin^2\mu) \\ &\quad - 2a_1\cos\mu[6g_1^2 - (a_0^2 - 2a_1^2)g_2], \\ d_4 &= 2a_2^2a_3^2(a_4^2 - 2a_2^2a_3^2 - 3a_1^2a_4^2) + (1 - 2a_0^2 - 2a_1^2 + 4a_0^2a_1^2 \\ &\quad - 2a_4^4)g_2 - 16a_1^2g_1^2\cos^2\mu + 2a_1g_1\cos\mu(5a_0^2 - 2 - 2a_0^4 \\ &\quad + a_1^2 - 3a_0^2a_1^2 + 2a_1^4 + 8a_2^2a_3^2 - 2a_0^2a_4^2 + 6a_1^2a_4^2), \\ d_5 &= 4(a_0^2 - a_1^2)g_1^2 + a_1^3g_1(g_3 - a_0^2)\cos 3\mu + a_1g_1\cos\mu[2a_0^2 \\ &\quad - (1 - 2a_0^2)^2 + a_1^2(g_3 + 4a_4^2 - 5a_0^2) + 4a_2^2a_3^2 - 8a_0^2a_4^2] \\ &\quad + 2a_1^2\cos 2\mu[(2a_0^2 - a_1^2)g_2 - 3g_1^2], \\ d_6 &= a_1\sin\mu\{g_1[2a_0^2(1 + 2a_2^2 + 2a_3^2 - 2a_4^2) - 1 + 4a_2^2a_3^2 \\ &\quad + 4a_1^2a_4^2 + 4a_1^2(g_3 - a_0^2)\cos^2\mu] + 4a_1[(2a_0^2 - a_1^2)g_2 \\ &\quad - 3g_1^2]\cos\mu\}, \\ d_7 &= g_1\{1 - 2a_0^4a_1^2 - 4a_2^2a_3^2 - 2a_4^2 + a_1^2(4a_4^2 - 2) \\ &\quad - a_0^2[2 + 2a_1^4 - 4a_4^2 - a_1^2(3 - 6a_4^2)] - 2a_1^2\cos^2\mu[2a_0^4 \\ &\quad + 3 - 12a_2^2a_3^2 - 4a_1^2(1 + a_4^2) + a_0^2(5a_1^2 - 7 + 2a_4^2)] \\ &\quad - 4a_1^3g_1\cos 3\mu\} + 2a_1\cos\mu\{(1 - 2a_0^2 - 2a_1^2 \\ &\quad + 3a_0^2a_1^2)g_2 - 2a_2^2a_3^2[2a_2^2a_3^2 - (1 + a_0^2 - 5a_1^2)a_4^2]\}, \\ d_8 &= 2a_1\sin\mu[(1 - 2a_0^2 - 2a_1^2 + 3a_0^2a_1^2)g_2 \\ &\quad - 2a_2^2a_3^2(2a_2^2a_3^2 - a_4^2 + a_0^2a_4^2 + 4a_1^2a_4^2) \\ &\quad - 4a_1^2g_1^2\cos 2\mu + a_1g_1\cos\mu(7a_0^2 - 3 - 2a_0^4 \\ &\quad + 4a_1^2 - 5a_0^2a_1^2 + 12a_2^2a_3^2 - 2a_0^2a_4^2 + 4a_1^2a_4^2)], \\ d_9 &= a_0^2a_1\{2a_1^2g_2\cos 3\mu - g_1[a_1\cos 2\mu(2a_0^2 - 3 + 3a_1^2 \\ &\quad + 6a_4^2) + a_1^3\cos 4\mu - 4g_1\cos\mu]\}, \\ d_{10} &= a_0^2a_1\{2a_1^2g_2\sin 3\mu - g_1[a_1\sin 2\mu(2a_0^2 - 3 + 3a_1^2 \\ &\quad + 6a_4^2) + a_1^3\sin 4\mu - 4g_1\sin\mu]\}. \end{aligned}$$

Note that $b_1 \neq 0$. By dividing the left side of (17) by the left side of (16), we obtain the polynomial remainder

$$-xk_1(\varphi) + k_2(\varphi) = 0, \quad (18)$$

where

$$\begin{aligned} k_1(\varphi) &= c_7 + c_5\cos 2\varphi + c_6\sin 2\varphi, \\ k_2(\varphi) &= c_1\cos 3\varphi + c_2\sin 3\varphi + c_3\sin\varphi + c_4\cos\varphi, \\ c_1 &= -b_2b_4d_1 + b_3b_5d_1 - b_1b_5d_2 + b_1b_4d_3 - b_1^2d_9, \\ c_2 &= -b_3b_4d_1 - b_2b_5d_1 - b_1^2d_{10} + b_1b_4d_2 + b_1b_5d_3, \\ c_3 &= b_3b_4d_1 - b_2b_5d_1 - b_1b_4d_2 + b_1b_5d_3 - b_1^2d_8 \\ &\quad + 2b_1b_3d_1a_0^2 - 2b_1^2d_2a_0^2, \end{aligned}$$

$$\begin{aligned}
c_4 &= -b_2b_4d_1 - b_3b_5d_1 + b_1b_5d_2 + b_1b_4d_3 - b_1^2d_7 \\
&\quad + 2b_1b_2d_1a_0^2 - 2b_1^2d_3a_0^2, \\
c_5 &= b_2^2d_1 - b_3^2d_1 - 4b_1b_4d_1 + b_1b_3d_2 - b_1b_2d_3 + 2b_1^2d_5, \\
c_6 &= 2b_2b_3d_1 - 4b_1b_5d_1 - b_1b_2d_2 - b_1b_3d_3 + 2b_1^2d_6, \\
c_7 &= b_2^2d_1 + b_3^2d_1 - b_1b_3d_2 - b_1b_2d_3 + 2b_1^2d_4 + 4b_1^2d_1a_0^2.
\end{aligned}$$

Using some algebra, we can prove that any φ such that $\sin \varphi = 0$ is not minimum point of $R(\theta, \varphi)$. Thus, we can suppose $\sin \varphi \neq 0$.

We first discuss the case $k_1(\varphi) \neq 0$. From (18), there is $x = \frac{k_2(\varphi)}{k_1(\varphi)}$. Substituting it in Eq.(16), we derive

$$\begin{aligned}
&V^6[2b_1(c_1 + c_4)^2 + b_2(c_5 + c_7)(c_1 + c_4) + (c_5 + c_7)^2(b_4 - b_1a_0^2)] \\
&+ V^5\{(c_1 + c_4)[4b_1(3c_2 + c_3) + 2b_2c_6 + b_3(c_5 + c_7)] \\
&+ (c_5 + c_7)[b_2(3c_2 + c_3) + 2b_5(c_5 + c_7) + 4c_6(b_4 - b_1a_0^2)]\} \\
&- V^4\{2b_1[8c_1^2 - (3c_2 + c_3)^2 - 2(c_1 - c_4)^2] - 2b_3(c_1 + c_4)c_6 \\
&+ 2(b_2c_1 - 4b_5c_6)(c_5 + c_7) + 2b_2(c_1c_5 - c_4c_7) \\
&- (c_5 - c_7)^2(b_4 + b_1a_0^2) - (3c_2 + c_3)(b_3c_5 + 2b_2c_6 + b_3c_7) \\
&+ 4b_4(c_5^2 - c_6^2) + 4b_1(c_6^2 + c_7^2)a_0^2\} \\
&- 2V^3[4b_1c_1(5c_2 + c_3) - (c_2 + c_3)(4b_1c_4 + b_2c_7) \\
&+ 2(b_3c_1 + b_2c_2)c_5 + 2b_5(c_5^2 - c_7^2) + b_2(3c_1 - c_4)c_6 \\
&- b_3(3c_2 + c_3)c_6 + b_3(c_1 - c_4)c_7 + 4c_6(b_4c_5 - b_5c_6 + b_1c_7a_0^2)] \\
&- V^2\{2b_1[(3c_1 - c_4)^2 - 8c_2^2 + 2(c_2 + c_3)^2] - 4b_3c_2c_5 \\
&+ (3c_1 - c_4)[b_2(c_5 - c_7) - 2b_3c_6] + 2c_7b_3(c_2 + c_3) \\
&+ 2b_2(c_3 - c_2)c_6 + 4b_4(c_5^2 - c_6^2) - 8b_5c_6(c_5 - c_7) \\
&+ (c_5 + c_7)^2(b_1a_0^2 - b_4) - 4b_1(c_6^2 + c_7^2)a_0^2\} \\
&+ V[(c_2 - c_3)(12b_1c_1 - 4b_1c_4 + b_2c_5 - 2b_3c_6 - b_2c_7) \\
&+ (c_5 - c_7)(3b_3c_1 - b_3c_4 + 2b_5c_5 + 4b_4c_6 - 2b_5c_7 + 4b_1c_6a_0^2)] \\
&= 0,
\end{aligned} \tag{19}$$

where $V = \cot \varphi$. Finding out the solution(s) (θ_i, φ_i) of (19) and (18) satisfying the two equations in (12) if there exists, then determining the minimum point (θ_1, φ_1) such that $\min\{R(\theta_i, \varphi_i)\} = R(\theta_1, \varphi_1)$ if there is, we obtain

$$R_{\min} = \begin{cases} \min\{R(\theta_1, \varphi_1), Q(\theta_0, \varphi_0), R(0, \varphi)\}, & \text{if } a_2 = a_3, \\ \min\{R(\theta_1, \varphi_1), R(0, \varphi)\}, & \text{if } a_2 \neq a_3. \end{cases}$$

Otherwise, there is no minimum point at the case $P(\theta, \varphi)Q(\theta, \varphi)\sin \theta \neq 0$, and the minimum

$$R_{\min} = \begin{cases} \min\{Q(\theta_0, \varphi_0), R(0, \varphi)\}, & \text{if } a_2 = a_3, \\ R(0, \varphi), & \text{if } a_2 \neq a_3. \end{cases} \tag{20}$$

Now let us look at the case $k_1(\varphi) = 0$. If it has common solution(s) with $k_2(\varphi) = 0$, and there is/are solutions/solutions (θ_j, φ_j) of Eqs.(16) and (17) satisfying Eqs.(12), then we have

$$R_{\min} = \begin{cases} \min\{R(\theta_2, \varphi_2), Q(\theta_0, \varphi_0), R(0, \varphi)\}, & \text{if } a_2 = a_3, \\ \min\{R(\theta_2, \varphi_2), R(0, \varphi)\}, & \text{if } a_2 \neq a_3, \end{cases}$$

where $R(\theta_2, \varphi_2) = \min\{R(\theta_j, \varphi_j)\}$. Otherwise, the minimum R_{\min} is the same as that in (20).

Note that in the three expressions of R_{\min} above, we use the properties $R(0, \varphi) = R(\pi, \varphi)$ and $Q(\theta_0, \varphi_0) = P(\theta_{0'}, \varphi_{0'})$, where $P(\theta_0, \varphi_0) = Q(\theta_{0'}, \varphi_{0'}) = 0$.

For the quantum channel (2) with $a_1a_2a_3a_4\sin \mu = 0$, we also obtain the exact values of the maximal successful probabilities for controlled teleportation.

According to the definition $E_{ij} = \max_{\epsilon} \sum_s p_s E(|\phi_s\rangle)$ of LE in [18], the maximal probability p_{\max} in (11) is a kind of LE. If $E(|\phi_s\rangle)$ is chosen to be the concurrence of $|\phi_s\rangle$, we show the exact value of the LE

$$\begin{aligned}
E_{23} &= \max\{p_1C_1 + p_2C_2\} \\
&= 2\sqrt{a_2^2a_3^2 - 2a_1a_2a_3a_4\cos \mu + (a_0^2 + a_1^2)a_4^2}.
\end{aligned}$$

In conclusion, we have shown the sufficient and necessary condition that a three-qubit state can be collapsed to an EPR pair by an appropriate measurement on one qubit. We also characterized the tripartite states that can be used for perfect controlled teleportation. Moreover, we gave the maximal successful probability for controlled teleportation via a general tripartite state, and determined the exact value of another localizable entanglement in [18].

The authors thank Prof. J. I. Cirac for his fruitful discussions and for his hospitality during their stay at Max-Planck-Institut für Quantenoptik. This work was supported by the National Natural Science Foundation (NSF) of China under Grant No: 10671054, Hebei NSF of China under Grant Nos: A2005000140, 07M006, and the Key Project of Science and Technology Research of Education Ministry of China under Grant No: 207011.

-
- [1] C. H. Bennett *et al.*, Phys. Rev. Lett. **70**, 1895 (1993).
 - [2] T. J. Johnson, S. D. Bartlett, and B. C. Sanders, Phys. Rev. A **66**, 042326 (2002).
 - [3] L. Vaidman, Phys. Rev. A **49**, 1473 (1994).
 - [4] S. L. Braunstein and H. J. Kimble, Phys. Rev. Lett. **80**, 869 (1998).
 - [5] D. Bruß *et al.*, Phys. Rev. A, **57**, 2368 (1998).
 - [6] A. Karlsson and M. Bourennane, Phys. Rev. A **58**, 4394 (1998).
 - [7] C. P. Yang, S. I. Chu, and S. Han, Phys. Rev. A **70**, 022329 (2004).
 - [8] A. K. Pati, Phys. Rev. A **61**, 022308 (2000).
 - [9] A. K. Pati and P. Agrawal, J. Opt. B: Quantum Semi-classical Opt. **6**, S844 (2004).
 - [10] F. G. Deng *et al.*, Phys. Rev. A **72**, 022338 (2005).
 - [11] D. Bouwmeester *et al.*, Nature (London) **390**, 575 (1997).
 - [12] M. A. Nielsen, E. Knill, and R. Laflamme, Nature (London) **396**, 52 (1998).
 - [13] M. Hillery, V. Bužek, and A. Berthiaume, Phys. Rev. A **59**, 1829 (1999).
 - [14] E. Biham, B. Huttner, and T. Mor, Phys. Rev. A **54**, 2651 (1996).
 - [15] P. D. Townsend, Nature **385**, 47 (1997).
 - [16] S. Bose, V. Vedral, and P. L. Knight, Phys. Rev. A **57**, 822 (1998).
 - [17] A. Acín *et al.*, Phys. Rev. Lett. **85**, 1560 (2000).
 - [18] F. Verstraete, M. Popp, and J. I. Cirac, Phys. Rev. Lett. **92**, 027901 (2004).